Worcester County Mathematics League
Varsity Meet 2 - December 2, 2020
Round 1 - Fractions, Decimals, and Percents

All answers must be in simplest exact form.

1. Simplify:

$$\frac{11}{12} - \frac{\frac{5}{4}}{2 - \frac{7}{2}} \cdot \frac{3}{4}$$

2. Express as a fraction in simplest form:

$$(0.\overline{349} + 0.\overline{267}) \div 0.\overline{426}$$

3. There are seven ounces of coffee in one mug and two ounces of cream in a second mug. Your math coach pours half of the coffee into the mug with the cream and stirs it until is completely mixed. She then pours half of the mixture into the first mug and stirs. What percentage of the liquid in the first mug is cream?

ANSWERS

(1 pt) 1. ______________________

(2 pts) 2. ______________________

(3 pts) 3. ______________________ percent

St. Peter-Marian, Monty Tech, QSC

Round 2 - Algebra I

All answers must be in simplest exact form.

1. Solve for x:
$$\frac{x}{5}+\frac{x+2}{3}=\frac{7x-1}{15}$$

2. Adam, Brenda, and Chris are bricklayers. Working together, Adam and Brenda can build a brick wall in 20 hours. Brenda and Chris together can build a wall in 15 hours. Adam and Chris together can build a wall in 12 hours. How many hours does it take Adam to build a wall by himself?

3. If $x+y=19$ and $xy=87$ then find x^3+y^3.

ANSWERS

(1 pt) 1. $x =$ ____________________

(2 pts) 2. ________________________ hours

(3 pts) 3. $x^3+y^3 =$ _______________

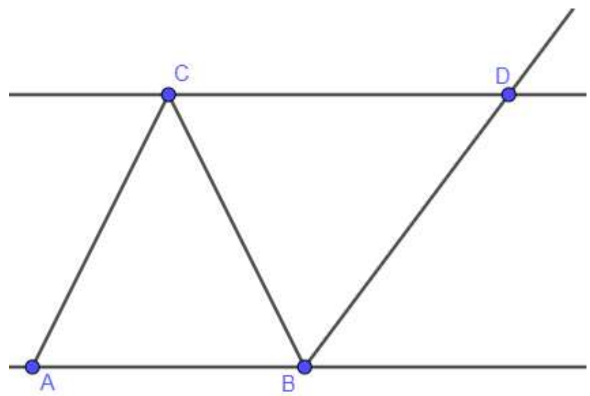
Bancroft, Leicester, Doherty

Worcester County Mathematics League
Varsity Meet 2 - December 2, 2020
Round 3 - Parallel Lines and Polygons

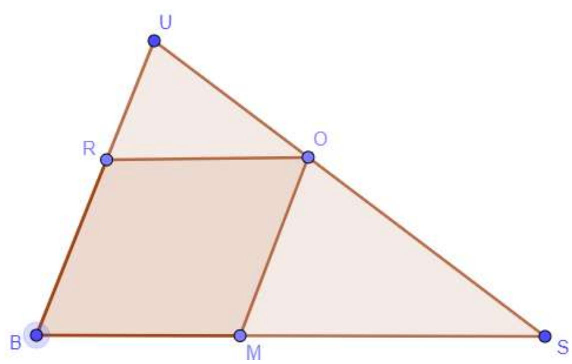
All answers must be in simplest exact form.

1. Each exterior angle of a regular polygon has measure 20°. How many sides does the polygon have?

2. Given the figure below where $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$; $AC = BC$; $m\angle CBD = 4 \cdot m\angle CDB$; $m\angle CAB = 35^\circ$. Find $m\angle CDB$.



3. The vertices of rhombus $ROMB$ lie on $\triangle BUS$. $BU = 10$ and $BS = 15$. Find BR.



ANSWERS

(1 pt) 1. ____________________

(2 pts) 2. ____________________ $^\circ$

(3 pts) 3. ____________________

Leicester, Bancroft, QSC

Worcester County Mathematics League

Varsity Meet 2 - December 2, 2020

Round 4 - Sequences and Series

All answers must be in simplest exact form.

1. The school auditorium has 20 rows of seats. The front row contains 25 seats. Each of the other rows has two more seats than the row in front of it. How many seats does the auditoriusm have in total?

2. The Maryam Mirzakhani fan club has 2020 members. Messages are communicated to the members of the club in the following manner. In the first round, the president and vice president each send the message to three members. In the second round, each of those six members sends the message to three different members. In subsequent rounds, each member who just received the message sends it to three more members. If the process continues this way, how many rounds of call are needed so that all 2020 members, including the president and vice president, receive the information in the message?

3. Find the sum of the following infinite series:

$$\frac{7}{2}+\frac{14}{2^2}+\frac{21}{2^3}+\frac{28}{2^4}+\cdots$$

ANSWERS

(1 pt) 1. ____________________

(2 pts) 2. ____________________

(3 pts) 3. ____________________

Tantasqua, Notre Dame Academy, Bancroft

Worcester County Mathematics League

Varsity Meet 2 - December 2, 2020

Round 5 - Matrices and Systems of Equations

All answers must be in simplest exact form.

1. If $|A|$ denotes the determinant of matrix A, find x given:

$$\begin{vmatrix} x-1 & 6 \\ 2 & x-2 \end{vmatrix} = 0$$

2. Larry, Mariam, and Nancy are friends and are going shopping. Nancy has \$15 more than Larry and Mariam combined. If Mariam gives \$4 to Larry, then Mariam will have one third as much money as Nancy. If, instead, Larry gives \$4 to Mariam, then Mariam will have twice as much money as Larry. What is the total amount of money of the three friends?

3. Find all values for w given the following system of equations and if $w = x + 2y + z$:

$$\begin{aligned} x + z &= 9 \\ x + 2y &= 6 \\ x^2 - 4yz &= -52 \end{aligned}$$

ANSWERS

(1 pt) 1. $x =$ ____________________

(2 pts) 2. \$ ____________________

(3 pts) 3. $w \in \{$____________________$\}$

Bancroft, Hudson, Shrewsbury

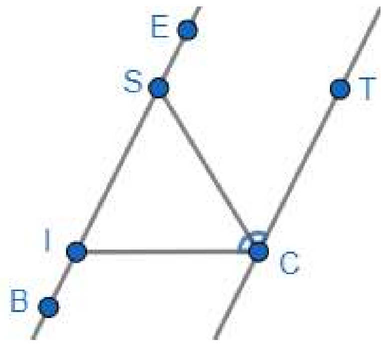
Worcester County Mathematics League
Varsity Meet 2 - December 2, 2020
Team Round

All answers must be in simplest exact form.

1. Mr. Young plans to bake a cake and some cookies. The cake requires 3/8 cups of sugar and the cookies require 3/5 cups of sugar. He has 13/16 cups of sugar. How much more sugar does Mr. Young need for his baking? Express your answer as a fraction $\frac{a}{b}$ in lowest terms.

2. Given that $ab = 12$, $bc = 48$, and $\sqrt{ac} = 4$, Find $a + b + c$.

3. In the figure below, $\overleftrightarrow{BE} \parallel \overleftrightarrow{CT}$, $m\angle BIC = 112°$, and $\angle ICS \cong \angle SCT$. Find $m\angle ESC$.



4. A female bee hatches from a fertilized egg and a male bee hatches from an unfertilized egg. Thus, a female bee has a male and female parent and a male bee has only a female parent. The number of ancestors of a male be follows a sequence 1, 2, 3, 5, 8, ... Find the next three terms in the sequence listed as an ordered triple (l, m, n).

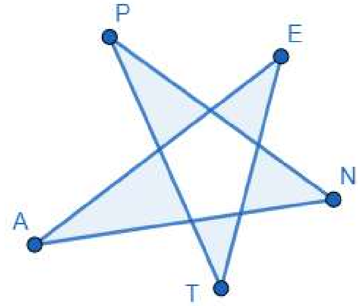
5. Solve the following equation for the matrix A.
$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} A = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix}$$

6. What percent of the first 30 positive integers have exactly four positive integer factors?

7. The expression $\sqrt{10 + \sqrt{10 + \sqrt{10 + ...}}}$ can be expressed in the form $\frac{a+\sqrt{b}}{c}$, where a, b, and c are postive integers with no common factors. Find the ordered triple (a, b, c).

8. Let $S_n = 1 + 3 + 5 + ... + a_n = 1234321$ where a_n is the n^{th} term in the sequence. Find n.

9. Find the sum of the measures of $\angle P, \angle E\angle N, \angle T$ and $\angle A$ in the figure below.



Shepherd Hill, Douglass, NDA, Wachusett, Tantasqua, Auburn, Westborough, Auburn, Tahanto

Worcester County Mathematics League
Varsity Meet 2 - December 2, 2020
Team Round Answer Sheet

ANSWERS

1. ______________________________

2. ______________________________

3. ______________________________

4. ______________________________

5. ______________________________

6. ______________________________

7. ______________________________

8. ______________________________

9. ______________________________

Worcester County Mathematics League
Varsity Meet 2 - December 2, 2020
Answer Key

Round 1 - Fractions, Decimals, and Percents

1. $\dfrac{37}{24}$
2. $\dfrac{308}{213}$
3. 16 or 16%

Round 2 - Algebra I

1. -11 or $x = -11$
2. 30 or 30 hours
3. 1900

Round 3 - Parallel Lines and Polygons

1. 18
2. 29 or 29° or 29 degrees
3. 6

Round 4 - Sequences and Series

1. 880
2. 6
3. 14

Round 5 - Matrices and Systems of Equations

1. $-2, 5$ (need both, either order) or $x \, \epsilon \{-2, 5\}$
2. \$93 or 93
3. $-13, 13$ (need both, either order) or $w \, \epsilon \{-13, 13\}$

Team Round

1. $\dfrac{13}{80}$
2. 16
3. 124°
4. $(13, 21, 34)$
5. $A = \begin{bmatrix} 3 & 5 \\ -1 & -1 \end{bmatrix}$
6. 30%
7. $(1, 41, 2)$
8. 1111
9. 180°

Wocomal Varsity Meet 2, Dec. 2, 2020. Solutions

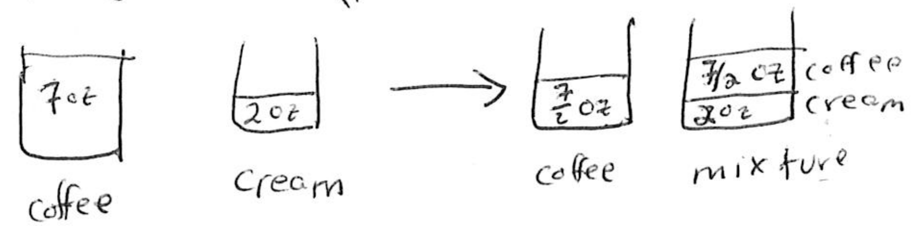
Round 1

1. $$\frac{11}{12} - \frac{\frac{5}{4}}{2-\frac{7}{2}}\cdot\frac{3}{4} = \frac{11}{12} - \frac{\frac{5}{4}}{-\frac{3}{2}}\cdot\frac{3}{4}$$
$$= \frac{11}{12} - \left(-\left(\frac{5}{4}\right)\left(\frac{2}{3}\right)\left(\frac{3}{4}\right)\right)$$
$$= \frac{11}{12} + \frac{5}{8}$$
$$= \frac{2\cdot 11}{2\cdot 12} + \frac{3\cdot 5}{3\cdot 8} = \frac{22+15}{24} = \boxed{\frac{37}{24}}$$

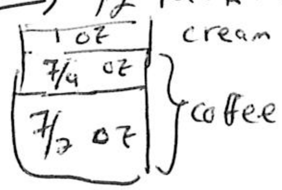
2. $$\left(.\overline{349} + .\overline{267}\right) \div .\overline{426} = (x+y)\div z$$

$$\begin{aligned} 1000x &= 349.\overline{349} \\ -\quad x &= .\overline{349} \\ \hline 999x &= 349 \\ x &= \frac{349}{999} \end{aligned} \qquad \begin{aligned} 1000y &= 267.\overline{267} \\ -\quad y &= .\overline{267} \\ \hline 999y &= 267 \\ y &= \frac{267}{999} \end{aligned} \qquad \begin{aligned} 1000z &= 426.\overline{426} \\ z &= .\overline{426} \\ \hline 999z &= 426 \\ z &= \frac{426}{999};\ \frac{1}{z} = \frac{999}{426} \end{aligned}$$

$$(x+y)\div z = \left(\frac{349}{999} + \frac{267}{999}\right)\frac{999}{426} = \frac{349+267}{426} = \frac{616}{426} = \boxed{\frac{308}{213}}$$

3. 

→ 1/2 of mixture is 7/4 oz coffee, 1 oz cream, or $\frac{11}{4}$ oz liquid



poured into first mug gives
1 oz cream in $1+\frac{7}{4}+\frac{7}{2} = \frac{25}{4}$ oz liquid.

or $\frac{1}{\frac{25}{4}} = \frac{4}{25} = \frac{16}{100} = \boxed{16\%}$

Wocomal Varsity Meet 2, round 2 SOLUTIONS Dec. 2, 2020

1. First, multiply both sides of the equation by LCD(3, 5, 15) = 15. Then solve the linear equation for x.

$$15\left(\frac{x}{5} + \frac{x+2}{3} = \frac{7x-1}{15}\right)$$

$$3x + 5(x+2) = 7x - 1$$

$$3x + 5x + 10 = 7x - 1$$

$$8x - 7x = -1 - 10$$

$$\boxed{x = -11}$$

2. Let
A = # of walls built by Adam working alone for 1 hour
B = # of walls built by Brenda working alone for 1 hour
C = # of walls built by Chris working alone for 1 hour

Then
$$\begin{cases} A + B = \frac{1}{20} \\ B + C = \frac{1}{15} \\ A + C = \frac{1}{12} \end{cases}$$

The system is solved by first eliminating B, and then eliminating C.

$$\left.\begin{array}{r} A + B = \frac{1}{20} \\ -\left(B + C = \frac{1}{15}\right) \end{array}\right\} \quad A - C = \frac{1}{20} - \frac{1}{15} = \frac{1}{5}\left(\frac{1}{4} - \frac{1}{3}\right) = \frac{1}{5}\left(\frac{3-4}{12}\right) = \frac{-1}{60}$$

$$\left.\begin{array}{r} A - C = \frac{-1}{60} \\ +\left(A + C = \frac{1}{12}\right) \end{array}\right\} \quad A - C + (A + C) = 2A = \frac{-1}{60} + \frac{1}{12} = \frac{-1}{60} + \frac{5}{60} = \frac{4}{60} = \frac{1}{15}$$

$$2A = \frac{1}{15} \Rightarrow A = \frac{1}{30}$$

Adam builds $\frac{1}{30}$ of a wall in one hour. Therefore it takes Adam $\boxed{30 \text{ hours}}$ to build a wall

Wocomal Varsity Meet 2, round 2 Solutions (cont.) Dec. 2, 2020

3. Note that: $x^3+y^3=(x+y)(x^2-xy+y^2)$

$$=(x+y)(x^2+2xy+y^2-3xy)$$

$$=(x+y)((x+y)^2-3xy)$$

Substituting $x+y=19$ and $xy=87$:

$$x^3+y^3=19((19)^2-3(87))$$

$$=19(361-261)$$

$$=19(100)$$

$$=\boxed{1900}$$

Wocomal Varsity Meet 2 Round 3 Dec. 2, 2020
Solutions.

1. The sum of the measures of the exterior angles is $360°$, and the exterior angles of a regular polygon are congruent; their measures are equal. If the number of sides is n, then the measure of an exterior angle is $360°/n = 20°$, or $n = \frac{360}{20} = \boxed{18}$

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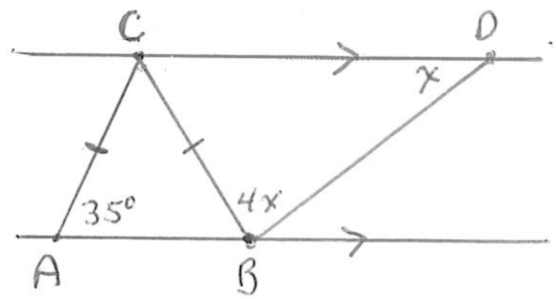
2. The diagram is shown at left with the given information marked and labeled. Let $m\angle CDB = x$; Then $m\angle CBD = 4x$, as labeled.



Note that ΔABC is isosceles, and base angles $\angle CAB \cong \angle CBA$.
Also $\angle BCD \cong \angle CBA$ because they are alternate interior angles created by transversal $\overleftrightarrow{BC}$ of parallel lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.
Therefore $\angle BCD \cong \angle CAB$, and $m\angle BCD = 35°$. The measures of the angles of ΔBCD sum to $180°$, so $35° + x + 4x = 180°$
Solving for x: $5x = 180 - 35 = 145°$, and $x = \boxed{29° = m\angle CDB}$

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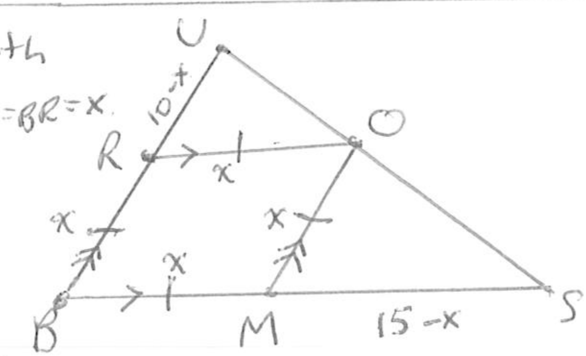
3. A rhombus is a quadrilateral with four congruent sides; Let $RO = OM = MB = BR = x$.
A rhombus is a parallelogram, so $\overline{BR} \parallel \overline{OM}$ and $\overline{RO} \parallel \overline{MB}$, as marked.



Then $\Delta OUR \sim \Delta SOM$ by AA similarity.
$\angle RUO \cong \angle MOS$ because they are corresponding angles created by transversal $\overleftrightarrow{US}$ of parallel lines $\overleftrightarrow{RB}$ and $\overleftrightarrow{OM}$ (cont.)

Wocomal Varsity Meet 2 round 3 Dec. 2, 2020

Solutions (cont.)

3. (cont.) Also $\angle OMS \cong \angle RBM$ because they are corresponding angles created by transversal $\overleftrightarrow{BM}$; and $\angle URO \cong \angle RBM$ because they are corresponding angles created by transversal $\overleftrightarrow{RB}$ of parallel lines $\overleftrightarrow{RO}$ and $\overleftrightarrow{MB}$. By transitivity, $\angle URO \cong \angle OMS$, and AA similarity can be applied to prove $\triangle OUR \sim \triangle SOM$. Therefore the following proportion is true: $\frac{UR}{RO} = \frac{OM}{MS}$.

Now $MS = BS - BM = 15 - x$ and $UR = BU - BR = 10 - x$, and $RO = OM = x$. Therefore $\frac{10-x}{x} = \frac{x}{15-x}$. Solving for x:

$x^2 = (10-x)(15-x) = 150 - 25x + x^2$; subtracting x^2:

$0 = 150 - 25x$; $25x = 150$ or $x = \boxed{6 = BR}$

Wocomal Varsity Meet 2 Round 4 Solutions Dec. 2, 2020

1. The total number of seats in the auditorium is equal to the series with 20 terms, first term $a_1=25$, and common difference $d=2$:

$$25+27+29+\ldots+\underbrace{(25+2\cdot 19)}_{63}=n\frac{(a_1+a_{20})}{2}=20\frac{(25+63)}{2}$$

$$=10(88)=\boxed{880}\text{ seats}$$

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2. The number of members contacted can be calculated with a geometric series with ratio 3 and first term 6:

$$2+(6+6\cdot 3+6\cdot 3\cdot 3+6\cdot 3^3+\ldots+6\cdot 3^{n-1})=2+6(1+3+3^2+\cdots+3^{n-1})$$

The formula for a geometric series of n terms starting with a_1 is: $S_n=a_1\frac{1-r^n}{1-r}$, where here $a_1=6$, $r=3$. Therefore the number of members contacted after n rounds is:

$$2+6\frac{1-3^n}{1-3}=2+\frac{6(3^n-1)}{2}=2+3^{n+1}-3=3^{n+1}-1$$

n	$3^{n+1}-1$
1	8
2	26
3	80
4	242
5	728
6	2186

Since for $n=6$, 2186 members can be contacted, and $2186>2020$, the number of rounds needed is $\boxed{6}$

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3. Let $x=\frac{7}{2}+\frac{14}{2^2}+\frac{21}{2^3}+\frac{28}{2^4}+\ldots$

Then $\frac{x}{2}=\frac{7}{2^2}+\frac{14}{2^3}+\frac{21}{2^4}+\ldots$

And $\frac{x}{2}=x-\frac{x}{2}=\frac{7}{2}+\frac{7}{2^2}+\frac{7}{2^3}+\frac{7}{2^4}+\ldots$ (subtracting the series for x/2 from the x series)

or $\frac{x}{2}=7(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\ldots)$

and $x=7(1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots)$

$=7\left(\frac{1}{1-\frac{1}{2}}\right)=7\frac{1}{(1/2)}=7\cdot 2=\boxed{14}$ Where the formula for an infinite series and $r=\frac{1}{2}$ was applied.

Wocomal Varsity Meet 2 Round 5 solutions Dec. 2, 2020

1. The determinant of a 2x2 matrix $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$.

Thus:
$$\begin{vmatrix} x-1 & 6 \\ 2 & x-2 \end{vmatrix} = (x-1)(x-2) - 6\cdot 2 = x^2 - 3x + 2 - 12$$
$$= x^2 - 3x - 10 = (x-5)(x+2) = 0$$

Thus $x - 5 = 0$ or $x = -2$; $x \in \{-2, 5\}$

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2. Let L = Larry's money (\$), M = Mariam's money, N = Nancy's money.

Then the given information is captured in the system of three equations:

$$L + M + \$15 = N$$
$$M - \$4 = \tfrac{1}{3}N$$
$$2(L - \$4) = M + \$4$$

Substitute N in the second equation, and drop "\$":

$$M - 4 = \tfrac{1}{3}(L + M + 15)$$

Multiply x 3: $3(M-4) = 3M - 12 = L + M + 15$

$$2M - L = 27$$

Rearrange the original third equation so that elimination can be applied to the above equation

$$2L - 8 = M + 4$$
$$-2(M - 2L = -12)$$
$$2M - L = 27$$

$$0 + 3L = 24 + 27 = 51$$
$$L = 17$$
$$2M = 27 + L = 27 + 17 = 44$$
$$M = 22$$
$$N = L + M + 15 = 17 + 22 + 15 = 54$$

$$L + M + N = 17 + 22 + 54 = \boxed{\$93}$$

Wocomal Varsity Meet 2 Round 5 solutions (cont.) Dec 2, 2020

3. To solve for w, note that $w = x + 2y + z = 6 + z$ by substitution of the second equation. So w can be found once z is found.

Next, eliminate y from the third equation using the second equation (again), rewritten as $y = \frac{6-x}{2} = 3 - \frac{x}{2}$:

$$x^2 - 4\left(3 - \frac{x}{2}\right)z = x^2 - (12 - 2x)z = -52$$
$$= x^2 + 2xz - 12z = -52 \quad (4)$$

Now square the first equation and subtract eqn. (4):

$$(x+z)^2 = x^2 + 2xz + z^2 = 81 \quad (=9^2)$$
$$-\left(x^2 + 2xz - 12z = -52\right)$$

$$z^2 + 12z = 133$$
$$\text{or } z^2 + 12z - 133 = 0$$
$$\text{or } (z+19)(z-7) = 0$$
$$z = -19 \text{ or } +7.$$

$w = z + 6$, so $w = -19 + 6 = -13$ or $w = 7 + 6 = 13$

$$\boxed{w \in \{-13, 13\}}$$

Wocomal Varsity Meet 2 Team Round Dec. 2, 2020

Solutions

1. Mr. Young needs $3/8 + 3/5$ cups of sugar and has $13/16$ cups.
He needs $13/16 - (3/8 + 3/5)$ more cups.

$$3/8 + \frac{3}{5} = 3\left(\frac{1}{8}+\frac{1}{5}\right) = 3\left(\frac{5+8}{40}\right) = 3\left(\frac{13}{40}\right) = \frac{39}{40}$$

$$\frac{39}{40} - \frac{13}{16} = \frac{2\cdot 39}{80} - \frac{5\cdot 13}{80} = \frac{78-65}{80} = \boxed{\frac{13}{80}}$$

---

2. $ab = 12,\ bc = 48,\ (\sqrt{ac})^2 = ac = 4^2 = 16$

$\frac{a}{c} = \frac{ab}{bc} = \frac{12}{48} = \frac{1}{4}$; $4a = c$; $ac = a(4a) = 4a^2 = 16 \Rightarrow a^2 = 4, a = 2$

$ac = 16 \Rightarrow 2c = 16$; $c = 8$. $bc = 48 \Rightarrow b(8) = 48$, $b = 6$

$a + b + c = 2 + 6 + 8 = \boxed{16}$ Note: If one of a, b, c are negative, then all three numbers are negative. $a + b + c = -2 + -6 + -8 = \boxed{-16}$

strictly speaking, $a+b+c = x$, $\boxed{x \in \{-16, 16\}}$ or $x = 16$ if a, b, c are positive

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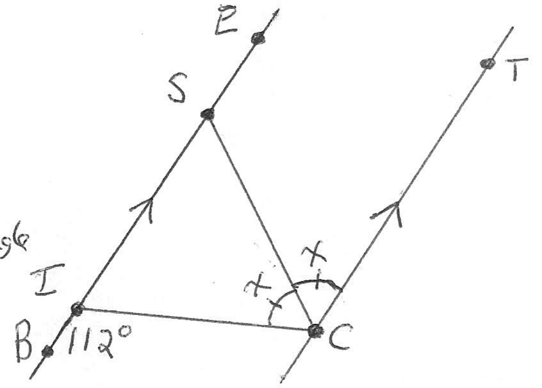
3. Let $m\angle ICS = m\angle SCT = x$. Then $\angle ISC$ and $\angle SCT$ are alternate interior angles and $\angle ISC \cong \angle SCT$ so $m\angle ISC = x$

Also $\angle BIC$ is an exterior angle of $\triangle ISC$, so

$m\angle BIC = 112° = m\angle ISC + m\angle ICS$ by the Exterior Angle Theorem.

$= x + x = 2x$



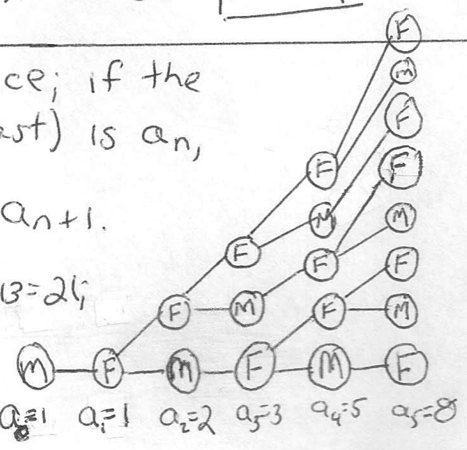
$2x = 112° \Rightarrow x = 56°$. $\angle ISC$ and $\angle ESC$ form a line, so $m\angle ESC = 180 - m\angle ISC = 180 - x = 180 - 56 = \boxed{124°}$

---

4. Note that sequence is the Fibonacci sequence; if the number of ancesters in the n-th generation (past) is a_n, then $a_1, a_2, a_3, \ldots = 1, 2, 3, \ldots$, and $a_{n+2} = a_n + a_{n+1}$.

Thus $a_4 = 5$; $a_5 = 8$; $a_6 = a_4 + a_5 = 5 + 8 = 13$; $a_7 = a_5 + a_6 = 8 + 13 = 21$;

$a_8 = a_6 + a_7 = 13 + 21 = 34$; and the next three terms are $\boxed{13, 21, 34}$



Wocomal Varsity Meet 2 Team Round Solutions (cont.) Dec 2, 2020

4. (cont.) The Fibonacci equation ($a_{n+2} = a_n + a_{n+1}$) can be justified given the parentage of bees. If a_n is the number of ancesters of the male bee n generations in the past, then the number of ancesters $(n+2)$ generations past is equal to the number of male ancesters plus twice the number of female ancesters in the $(n-1)^{th}$ generation past. Now the number of female ancesters in the $(n-1)^{th}$ generation is equal to a_n, the total number of ancesters in the n^{th} generation past, so $a_{n+2} = a_n + a_{n+1}$.

5. $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} A = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix}$; Multiply both sides by $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}^{-1}$: $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix}$

The inverse of 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is equal to: $\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, so

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}^{-1} = \frac{1}{1\cdot7 - 3\cdot2}\begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} = \frac{1}{7-6}\begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$$

Then $A = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix} = \boxed{\begin{bmatrix} 3 & 5 \\ -1 & -1 \end{bmatrix}}$

6. If $m = pq$, where p and q are prime numbers, then m has four factors: $1, p, q$, and $pq = m$. If $m = p^3$ (cube of a prime #) then m has four factors: $1, p, p^2$, and $p^3 = m$. All other prime factorizats have a number of factors different from four.
Count the number of integers less than 30 that are a product of two primes (there are 7): $2\cdot3 = 6$, $2\cdot5 = 10$, $2\cdot7 = 14$, $2\cdot11 = 22$, $2\cdot13 = 26$, $3\cdot5 = 15$, $3\cdot7 = 21$
There are two cubes of primes less than 30: $2^3 = 8$, $3^3 = 27$
$7 + 2 = 9$, therefore 9 of the first 30 positive integers have four factors.
$9/30 = 3/10 = \boxed{30\%}$

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7. Let $x = \sqrt{10+\sqrt{10+\sqrt{10+\ldots}}}$

Then $x^2 = 10 + \sqrt{10+\sqrt{10+\ldots}} = 10 + x$, or $x^2 = 10 + x$

Subtract $x+10$ from both sides: $x^2 - x - 10 = 0$

Apply the quadratic formula ($x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ for $ax^2+bx+c=0$):

$$x = \frac{1 \pm \sqrt{(-1)^2+4(10)}}{2} = \frac{1 \pm \sqrt{41}}{2}.$$ $x>0$, so $\frac{1-\sqrt{41}}{2} < 0$ is extraneous.

Therefore $x = \frac{1+\sqrt{41}}{2} = \frac{a+\sqrt{b}}{c}$, and $(a,b,c) = \boxed{(1,41,2)}$

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8. $S_n = 1+3+5+\ldots+a_n$; S_n is the sum of the first n odd integers.

It is an arithmetic series with $a_1=1$, $d=2$, and $a_n = 2n-1$.

Apply the formula for an arithmetic series: $S_n = n\frac{(1+2n-1)}{2} = n\frac{2n}{2} = n^2$.

Note that $1234321 = (1111)^2$. Therefore $S_n = n^2 = (1111)^2$ and $\boxed{n = 1111}$

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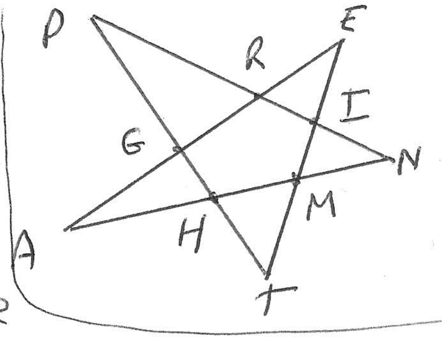
9. First label the points of intersection in the figure G, R, I, M, H, and note that they are the vertices of pentagon GRIMH. The sum of its interior angles ($\angle TGE$, $\angle ARN$, $\angle PIT$, $\angle EMA$, and $\angle PHN$) is $(5-2)180° = 3(180°) = 540°$. Now, the sum of the measures of the interior angles of $\triangle TGE$, $\triangle ARN$, $\triangle PIT$, $\triangle EMA$, and $\triangle PHN$ is $5 \cdot 180 = 900°$ because the interior angles of a triangle is 180° and there are 5 triangles. Writing this sum:



$m\angle T + m\angle TGE + m\angle E + m\angle A + m\angle ARN + m\angle N + m\angle P + m\angle PIT + m\angle T + m\angle E + m\angle EMA + m\angle A + \ldots$
$\ldots + m\angle P + m\angle PHN + m\angle N = 900$

Rearranging, and noting that each of the angles $\angle P, \angle E, \angle N, \angle T, \angle A$ appears twice in the sum:

$2(m\angle T + m\angle A + m\angle N + m\angle P + m\angle E) + (m\angle TGE + m\angle ARN + m\angle PIT + m\angle EMA + m\angle PHN)$

$= 2x + 540 = 900$; $2x = 900 - 540 = 360$; $x = \frac{360}{2} = \boxed{180°}$